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Working With Sets

Unit 3 Lesson 5

WORKING WITH SETS

Students will be able to:

use set-builder notation and/or roster form
to illustrate the elements of a set, and

find the complement of a subset of a given set,
within a given universal set.

WORKING WITH SETS

Key Vocabulary:

- Set
- Set-Builder Notation
- Null Set
- Universal Set
- Roster Form
- Subset
- Empty Set
- Complement of a Set

WORKING WITH SETS

SET is a collection of elements or members. Use braces, { }, to denote a set.

ROSTER FORM lists the elements of a set within braces.

Example:

A. The set, S, that contains the elements 1,2,3, and 4 when written in roster form is:

$$S = \{1, 2, 3, 4\}$$

B. The set, N, than contains all natural numbers when written in roster form is

$$N = \{1, 2, 3, 4, 5, 6, \ldots\}$$

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SET-BUILDER NOTATION describes elements of a set. It uses a variable and limits, or conditions, on the variable.

Example:

A. The set $S = \{1, 2, 3, 4\}$ when written in set-builder notation is:

$$S = \{x \mid x \text{ is a natural number and } x < 5\}$$

and read as: "the set of all values of x such that x is natural number and less than 5"

B. The set $N = \{1, 2, 3, 4, 5, 6, \ldots\}$ when written in set-builder notation is:

$$N = \{x \mid x \text{ is a natural number}\}$$

and read as: "the set of all values of x such that x is natural number"

WORKING WITH SETS

Sample Problem 1: Write each set in Roster Form and Set-Builder Notation.

A. $\boldsymbol{M}$ is the set of even whole number less than 13

B. $\boldsymbol{N}$ is the set of natural number greater than or equal to 11

C. $\boldsymbol{R}$ is the set of negative whole number less than -3

D. $\boldsymbol{S}$ is the set of even prime number

WORKING WITH SETS

Sample Problem 1: Write each set in Roster Form and Set-Builder Notation.

A. M is the set of even whole number less than 13

$M = \{2, 4, 6, 8, 10, 12\}$ $M = \{x \mid x \text{ is an even whole number and } x < 13\}$

B. N is the set of natural number greater than or equal to 11

$N = \{11, 12, 13, \ldots\}$ $N = \{x \mid x \text{ is a natural number and } x \geq 11\}$

C. R is the set of negative whole number less than -3

$R = \{2, 4, 6, 8, 10, 12\}$ $R = \{x \mid x \text{ is an even whole number and } x < 13\}$

D. S is the set of even prime number

$S = \{2\}$ $S = \{x \mid x \text{ is an even prime number}\}$

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Sample Problem 2: Write the solutions of each inequality in set-builder notation.

A. $\boldsymbol{2x + 2 < 24}$

B. $\boldsymbol{-(3x + 5) \leq -23}$

C. $\boldsymbol{3(x - 5) > 10 - 6x}$

D. $\boldsymbol{-3(x + 7) \geq 2x - 16}$

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Sample Problem 2: Write the solutions of each inequality in set-builder notation.

A. $2x + 2 < 24$

$$\begin{aligned} 2x + 2 &< 24 \\ 2x + 2 - 2 &< 24 - 2 \\ 2x &< 22 \\ \frac{2x}{2} &< \frac{22}{2} \\ x &< 11 \end{aligned}$$

$\{x \mid x < 11\}$

B. $-(3x + 5) \leq -23$

$$\begin{aligned} -(3x + 5) &\leq -23 \\ -3x - 5 + 5 &\leq -23 + 5 \\ -3x &\leq -18 \\ \frac{-3x}{-3} &\geq \frac{-18}{-3} \\ x &\geq 6 \end{aligned}$$

$\{x \mid x \geq 6\}$

Sample Problem 2: Write the solutions of each inequality in set-builder notation.

C. $3(x-5) > 10 - 6x$

$$\begin{aligned}
3(x-5) &> 12 - 6x \\
3x - 15 &> 12 - 6x \\
3x - 15 + 15 &> 12 + 15 - 6x \\
3x &> 27 - 6x \\
3x + 6x &> 27 - 6x + 6x \\
9x &> 27 \\
\frac{9x}{9} &> \frac{27}{9} \\
x &> 3
\end{aligned}$$

$\{x \mid x > 3\}$

D. $-3(x+7) \geq 2x - 16$

$$\begin{aligned}
-3(x+7) &\geq 2x - 16 \\
-3x - 21 &\geq 2x - 16 \\
-3x + 3x - 21 &\geq 2x + 3x - 16 \\
-21 &\geq 5x - 16 \\
-21 + 16 &\geq 5x - 16 + 16 \\
-5 &\geq 5x \\
-\frac{5}{5} &\geq \frac{5x}{5} \\
-1 &\geq x
\end{aligned}$$

$\{x \mid -1 \geq x\}$

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Sample Problem 2: Write the solutions of each inequality in set-builder notation.

A. $\boldsymbol{2x + 2 < 24}$

B. $\boldsymbol{-(3x + 5) \leq -23}$

C. $\boldsymbol{3(x - 5) > 10 - 6x}$

D. $\boldsymbol{-3(x + 7) \geq 2x - 16}$

WORKING WITH SETS

SUBSET is consists of elements from any given set.

NULL SET or **EMPTY SET** is a set that contains no elements. It is a subset of every set. Use { } or ∅ to represent the null set.

Example: If $A = \{3, 6, 9\}$ then its subsets are

Null set	$\{\ \}$
With one element	$\{3\}$ $\{6\}$ $\{9\}$
With two element	$\{3, 6\}$ $\{3, 9\}$ $\{6, 9\}$
With the original set	$\{3, 6, 9\}$

The 8 subsets of $A = \{3, 6, 9\}$ are

$\{\ \}, \{3\}, \{6\}, \{9\}, \{3, 6\}, \{6, 9\}, \{3, 9\},$ and $\{3, 6, 9\}.$

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Sample Problem 3: List all possible subsets of each given set.

A. $\boldsymbol{J = \{7, 11\}}$

B. $\boldsymbol{K = \{5, 9, 18\}}$

C. $\boldsymbol{D = \{-1, 9\}}$

D. $\boldsymbol{F = \{-5, -2, 1\}}$

WORKING WITH SETS

Sample Problem 3: List all possible subsets of each given set.

A. $J = \{7, 11\}$

The 4 subsets are $\{\quad\}, \{7\}, \{11\}$, and $\{7, 11\}$.

B. $K = \{5, 9, 18\}$

The 8 subsets are $\{\quad\}, \{5\}, \{9\}, \{18\}, \{5, 9\}, \{5, 18\}, \{9, 18\}$, and $\{5, 9, 18\}$.

C. $D = \{-1, 9\}$

The 4 subsets are $\{\quad\}, \{-1\}, \{9\}$, and $\{-1, 9\}$.

D. $F = \{-5, -2, 1\}$

The 8 subsets are $\{\quad\}, \{-5\}, \{-2\}, \{1\}, \{-5, -2\}, \{-5, 1\}, \{-2, 1\}$, and $\{-5, -2, 1\}$.

WORKING WITH SETS

UNIVERSAL SET is the largest set that consists of all elements from the given set.

COMPLEMENT OF A SET is a set that contains the elements of a universal set not contained in a given subset. Use $\boldsymbol{A'}$ to represent the complement of set $\boldsymbol{A}$.

Example: If $\boldsymbol{A = \{4, 8, 12, 20, 24, 28\}}$ is the universal set and $\boldsymbol{B = \{20, 24, 28\}}$. Find the $\boldsymbol{B'}$.

$$\boldsymbol{B' = \{4, 8, 12\}}$$

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Sample Problem 4:

A. Given $\boldsymbol{A \subseteq B}$, $\boldsymbol{B = \{1, 2, 4, 8, 16, 32\}}$, and $\boldsymbol{A = \{2, 8, 32\}}$. Find $\boldsymbol{A'}$.

B. Given $\boldsymbol{J \subseteq K}$, $\boldsymbol{J = \{-3, 5, 9, 17, 21\}}$, and $\boldsymbol{K = \{-3, 1, 5, 9, 13, 17, 21, 25\}}$. Find $\boldsymbol{J'}$.

C. Given $\boldsymbol{E \subseteq D}$, $\boldsymbol{D = \{-10, -8, -4, 2, 10, 20, 32\}}$, and $\boldsymbol{E = \{-8, 10\}}$. Find $\boldsymbol{E'}$.

D. Given $\boldsymbol{X \subseteq Y}$, $\boldsymbol{Y = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}}$, and $\boldsymbol{X = \{2, 5, 7, 13, 17, 19\}}$. Find $\boldsymbol{X'}$.

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Sample Problem 4:

A. Given $A \subseteq B$, $B = \{1, 2, 4, 8, 16, 32\}$, and $A = \{2, 8, 32\}$. Find A'.

$A' = \{1, 4, 16\}$

B. Given $J \subseteq K$, $J = \{-3, 5, 9, 17, 21\}$, and $K = \{-3, 1, 5, 9, 13, 17, 21, 25\}$. Find J'.

$J' = \{1, 13, 25\}$

C. Given $E \subseteq D$, $D = \{-10, -8, -4, 2, 10, 20, 32\}$, and $E = \{-8, 10\}$. Find E'.

$E' = \{-10, -4, 2, 20, 32\}$

D. Given $X \subseteq Y$, $Y = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, and $X = \{2, 5, 7, 13, 17, 19\}$. Find X'.

$X' = \{3, 11, 23\}$